

On your index card please tell us

- Your name and email address if you would like links to our presentation and website.
- The grade(s) and subject(s) you teach
- What is Differentiated Instruction (DI), to you?
- What elements of DI do you currently include in your teaching?
- What do you consider the most challenging aspect of DI?

LESSONS TO DIFFERENTIATE INSTRUCTION FOR MIDDLE SCHOOL STUDENTS

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The IDR²eAM Project

- Investigating Differentiated Instruction and Relationships between Rational Number Knowledge and Algebraic Reasoning in Middle School
- *Purposes of IDR²eAM:*
 - To investigate how to differentiate mathematics instruction in middle school for students with diverse cognitive characteristics
 - To understand relationships between students' rational number knowledge and algebraic reasoning
 - To build a community of educators interested in exploring how to differentiate mathematics instruction for middle school students

IDR²eAM Timeline

- Years 1 & 2 (2013-2015):
 - Conducted three 18-session after-school math classes with cognitively diverse groups of students.
 - Analyzed data to understand student thinking and features of differentiation.
- Year 3 (2015-2016): Teacher Study Group
 - Gathered 15 teachers from across the state to learn together about differentiating instruction in middle school math classes.
 - 3-day introductory workshop on DI, monthly meetings, 1-day final meeting to present their work.
 - You will see three of those final presentations today.
- Years 4 & 5 (2016-2018): Researcher/practitioner co-teaching in full classrooms.

What is Differentiated Instruction, for us?

- DI is proactively tailoring instruction to students' mathematical thinking while aiming to develop a cohesive classroom community (cf. Tomlinson, 2005)
 - Posing problems in harmony with and at the edge of students' thinking (Hackenberg, 2010), which relies on on-going formative assessment (Heacox, 2002)
 - Interacting responsively during class meetings (Jacobs & Empson, 2015)
 - Seeking to use thinking from individuals and small groups to shape whole classroom interactions (cf. Tomlinson, 2005)

Need:
Strategies for
asking
questions and
posing problems

**Getting
to know
student
thinking**

Need:
Strategies for
giving students
choices and for
tiering
instruction

Need:
Strategies for
making and
organizing
interpretations
of student
thinking

Some DI Strategies

- Lower prep:
 - Small group check-in and instruction
 - Open Questions
 - Choice Questions
 - Number Talks
 - Student-teacher goal-setting
 - Varied supplemental materials
 - Giving variable amounts of time for tasks, assessment
 - Check ins: Fist to 5, thumbs, highlighter colors
- Higher prep:
 - Open-ended problems and request for two solutions
 - Open Questions
 - Choice Questions and Parallel Tasks
 - Tiered Instruction
 - Learning Contracts

DIFFERENTIATED LESSON: DECOMPOSING SHAPES

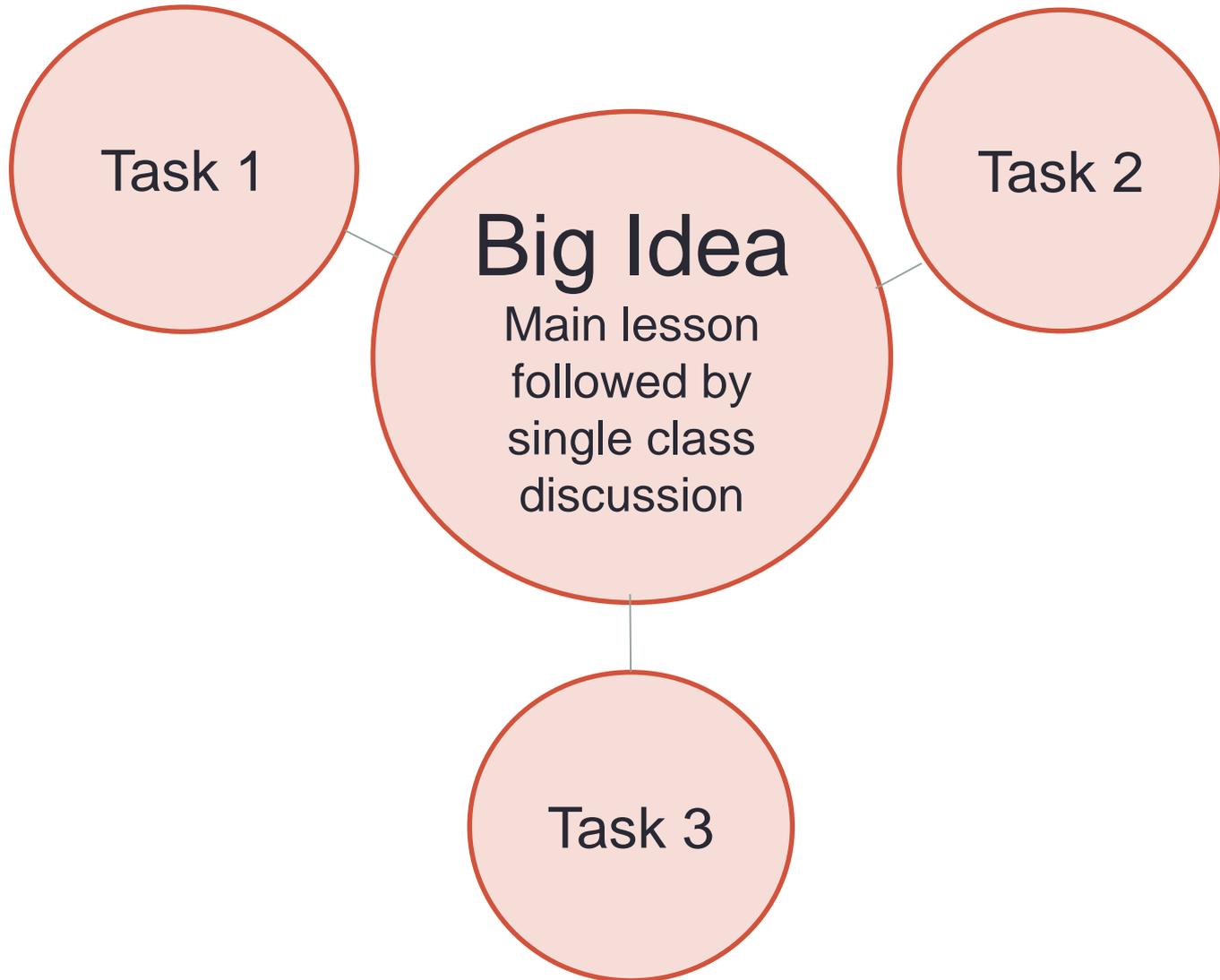
Anita Mendis

St. Therese Little Flower Catholic School

Middle School Math and Science

Student Thinking and Parallel Tasks

Parallel Tasks



Parallel tasks

- Sets of two or more related tasks that explore the same big idea.
- Tasks are designed to suit the needs of students at different learning levels.
- Tasks are similar so that all students can participate in a single follow up discussion.

Big Idea: Decomposing Shapes

Standard

6.GM.4: Find the area of complex shapes composed of polygons by composing or decomposing into simple shapes; apply this technique to solve real-world and other mathematical problems.

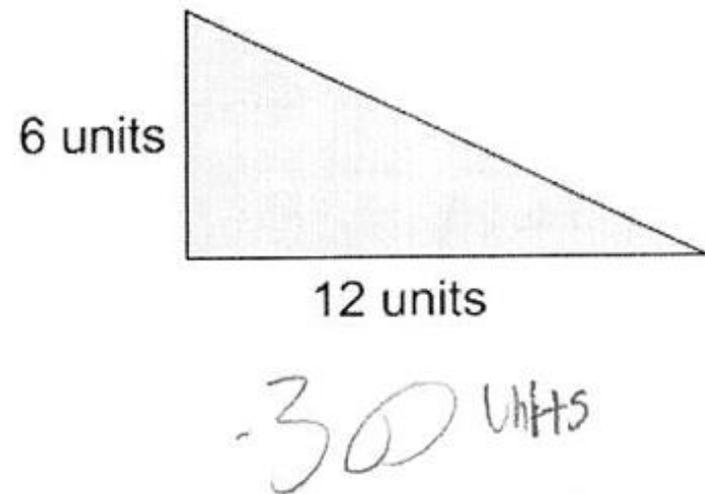
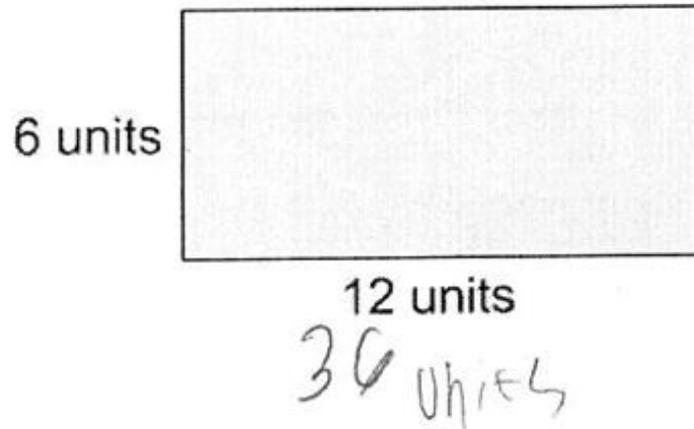
Follow up questions:

- If you cut up a shape and rearrange the pieces, does the total area change?
- Why might it be useful to cut up a shape to determine its area?
- How else could you cut up your shape to make it easy to relate the area of your shape to the area of another shape with an easy to calculate area?

Pre- Assessment

Student Thinking: Confused Perimeter and Area

Find the area of the shapes below:



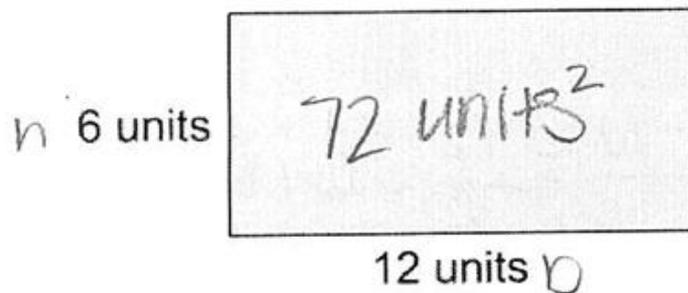
Are the formulas or the way that you find the areas of these shapes related in any way?

Yes because on this one all you had to
do was take away one six and I found the area

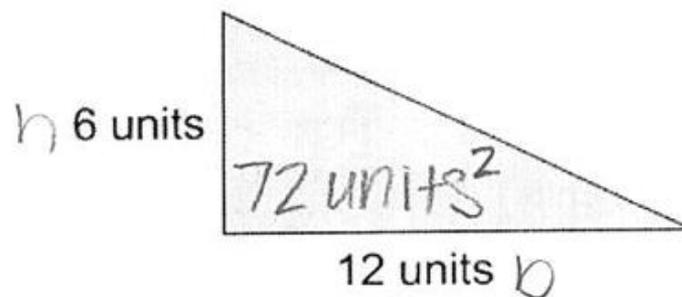
Pre-Assessment

Student Thinking: Area is the same for basic shapes with same dimensions

Find the area of the shapes below:



$$12 \cdot 6 = 72 \text{ units}^2$$



$$12 \cdot 6 = 72 \text{ units}^2$$

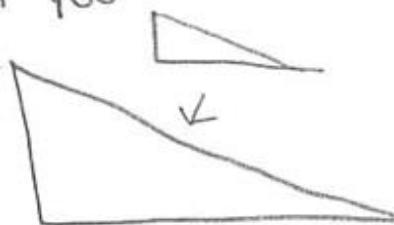
Can you use the area of one shape to find the area of the other shape?

Yes, because if they have the same base and height you could get the same area.

Are the formulas or the way that you find the areas of these shapes related in any way? **Yes**

the math part of it

When you add to shapes together



YO Add numbers as well

$$6 \times 7 = 72$$



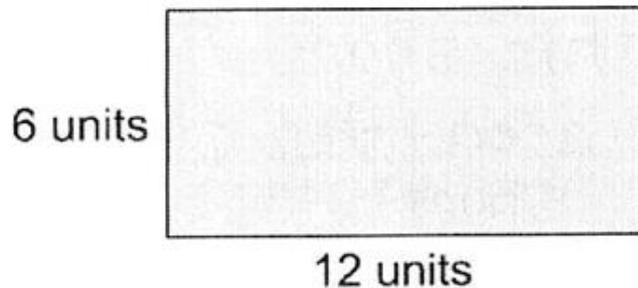
$$6 \times 7 = 72$$

$$= \begin{array}{r} 72 \\ + 72 \\ \hline 144 \end{array}$$

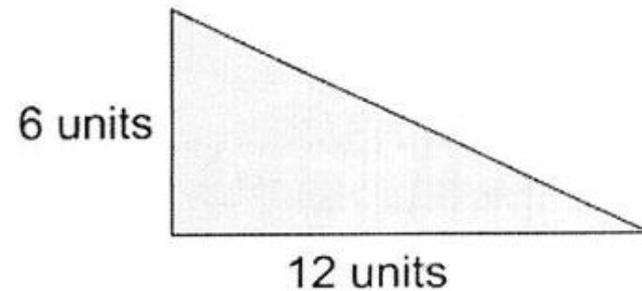


Pre-Assessment

Student Thinking: Understood relationship between basic shapes



$$\begin{array}{r} 1 \\ \times 12 \\ \hline 6 \\ \hline \end{array} \text{units}^2$$



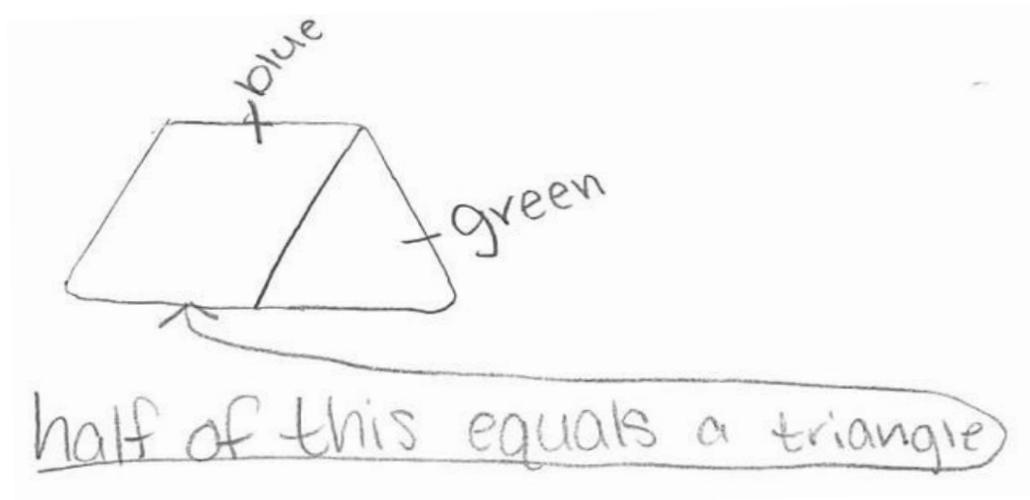
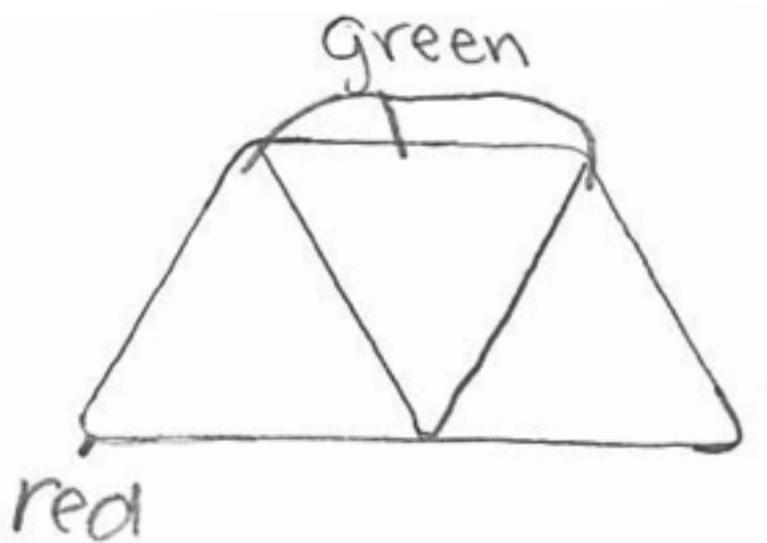
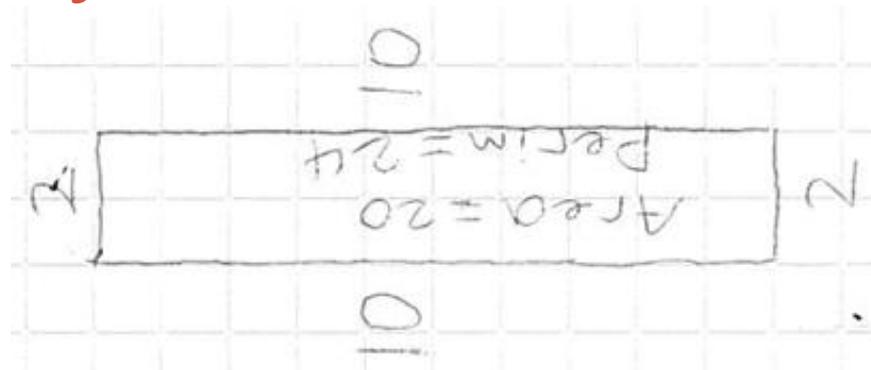
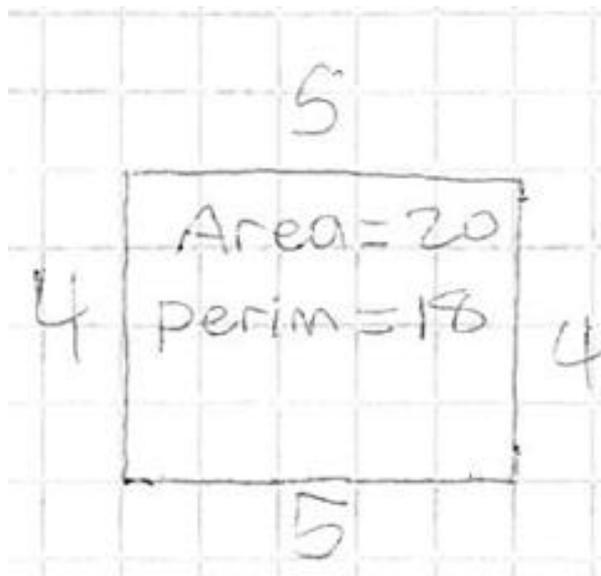
$$\begin{array}{r} 1 \\ \times 12 \\ \hline 6 \\ \hline \end{array} \text{units}^2$$
$$2 \sqrt{\begin{array}{r} 72 \\ -6 \vee \\ \hline 12 \\ -12 \\ \hline 0 \end{array}} \text{units}^2$$

Task 1: On graph paper draw as many different rectangles as you can with a total square area of 20. Compare their perimeters.

Task 2: How can dividing the red trapezoid pattern block into two shapes show that its area is $\frac{3}{2}$ of the area of the blue rhombus pattern block?

Decomposing shapes

Student Thinking Day 2:



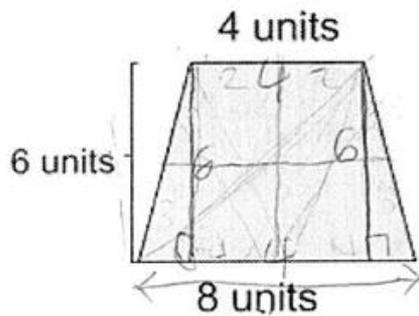
```
graph TD; A((Decomposing shapes)) --- B(Task 1: Find the area of the trapezoid); A --- C(Task 2: Find the area of the irregular shape);
```

Task 1: Find the area of the trapezoid

Task 2: Find the area of the irregular shape

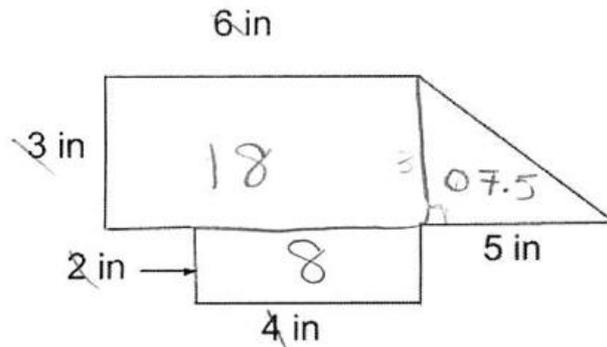
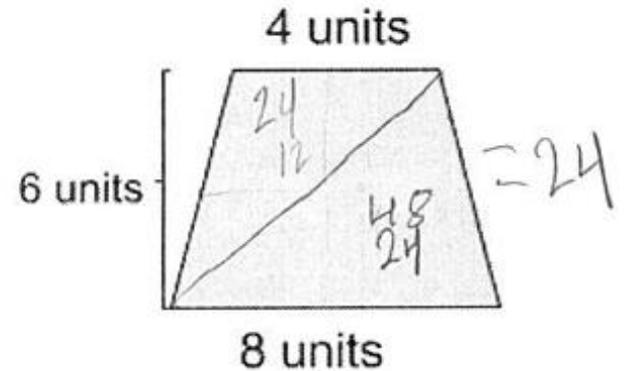
Decomposing shapes

Student Thinking Day 3: Some students treated complex shapes as the closest shape familiar to them. Others were decomposing but not in a way that made mathematical sense.



$$\begin{array}{r} \text{area } \square = 24 \\ \times 2 \\ \hline 36 \text{ units} \end{array}$$

$$\begin{array}{r} \text{area } \triangle = 6 \\ + 6 \\ \hline 36 \text{ unit} \end{array}$$



area = 33.5 in²

~~$$\begin{array}{r} 6 \\ \times 3 \\ \hline 18 \\ \times 2 \\ \hline 36 \\ \times \\ \hline 144 \\ \times 2 \\ \hline 288 \\ \times 5 \\ \hline 1440 \end{array}$$~~

$$\begin{array}{r} 07.5 \\ 2 \overline{) 15.0} \\ \underline{-06} \\ 15 \\ \underline{-14} \\ 10 \end{array}$$

$$\begin{array}{r} 18 \\ + 8 \\ \hline 26.0 \\ + 7.5 \\ \hline 33.5 \end{array}$$

```
graph TD; A((Decomposing shapes)) --- B((Task 1: Draw the shapes on graph paper and count their square area.)); A --- C((Task 2: Find the area of the irregular shape)); A --- D((Task 3: Find the area of the complex shape));
```

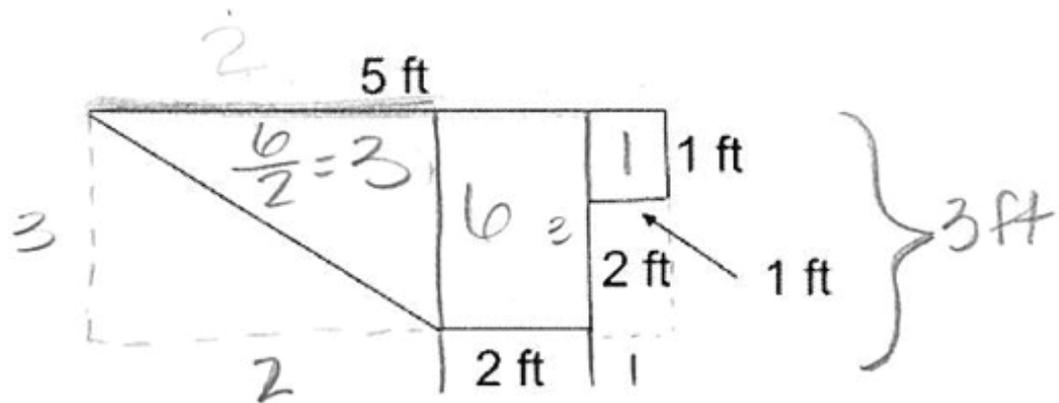
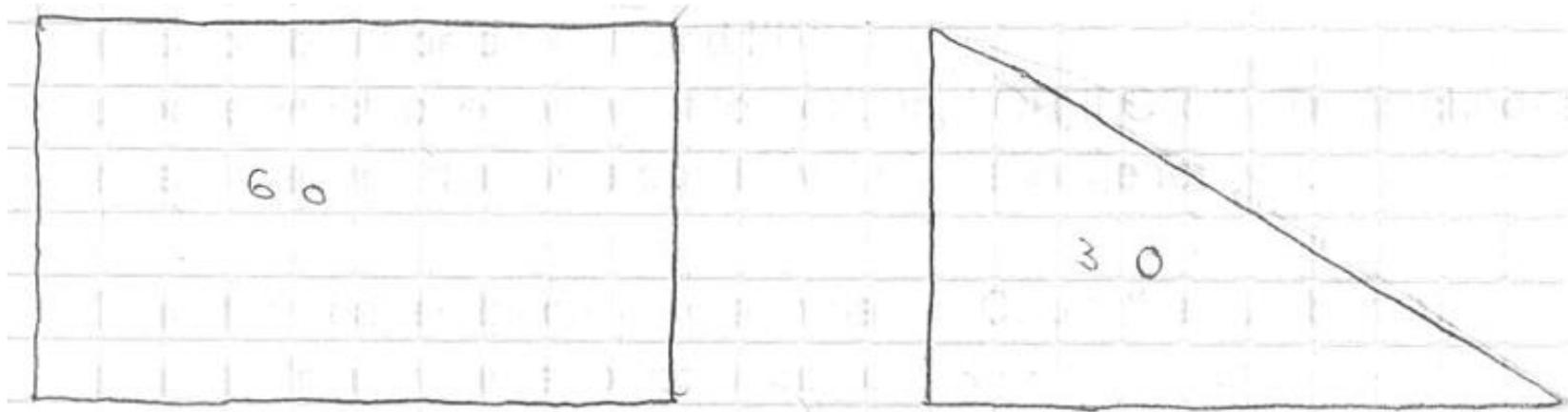
Task 1: Draw the shapes on graph paper and count their square area.

Task 2: Find the area of the irregular shape

Decomposing shapes

Task 3: Find the area of the complex shape

Student Thinking Day 4: Review & Practice

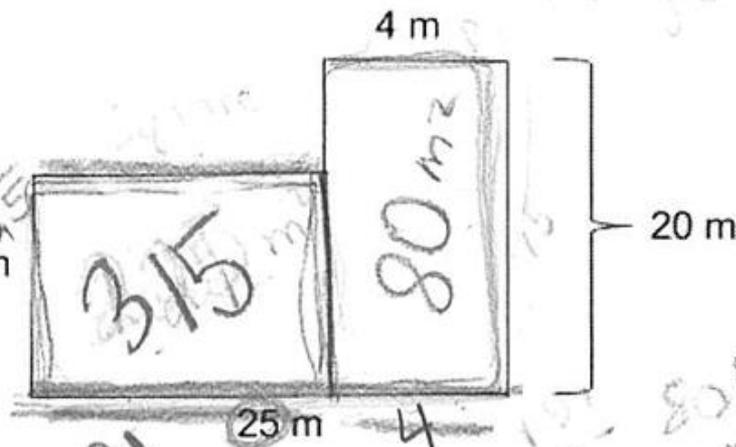


$$\begin{array}{r} \cancel{7.5} \\ \cancel{2 \overline{) 15.0}} \\ \cancel{- 10} \\ \hline \cancel{5} \end{array}$$
$$\begin{array}{r} 6 \\ + 3 \\ \hline 9 \\ + 1 \\ \hline 10 \end{array}$$

$$\text{total area} = 10^2 \text{ ft}$$

Student Thinking Day 4: Decomposing shapes but not dimensions

IV. Find the area of the irregular shape below:



III. Find the area of the complex shape below:

Handwritten calculations and notes:

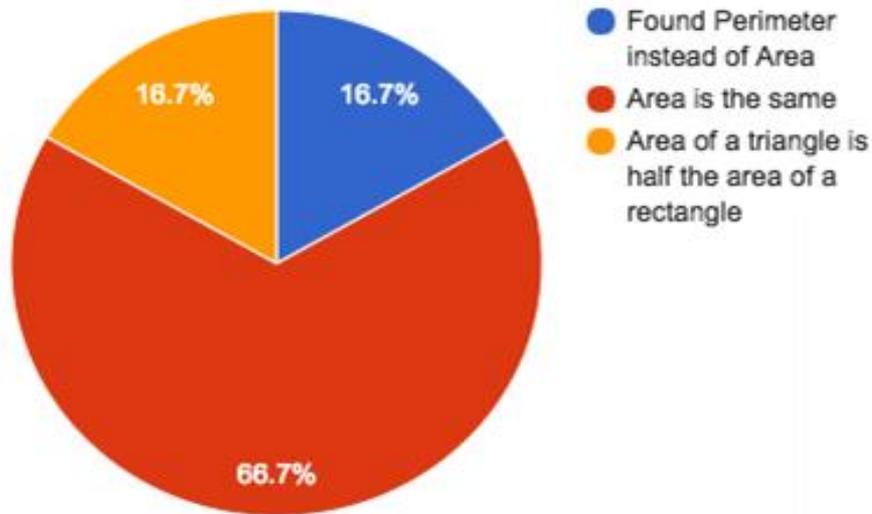
- $$\begin{array}{r} 25 \\ \times 15 \\ \hline 125 \\ + 250 \\ \hline 375 \end{array}$$
- $$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ + 150 \\ \hline 225 \end{array}$$
- $$\begin{array}{r} 20 \\ \times 4 \\ \hline 80 \end{array}$$
- $$\begin{array}{r} 80 \\ \overline{)375} \\ 320 \\ \hline 55 \end{array}$$
- $$\begin{array}{r} 315 \\ \sqrt{225} \\ - 240 \\ \hline 095 \\ - 80 \\ \hline 15 \end{array}$$
- $$21 \times 19 = 399$$
- $$21 \times 18 = 378$$
- $$21 \times 17 = 357$$
- $$21 \times 16 = 336$$
- $$21 \times 15 = 315$$
- $$21 \times 14 = 294$$
- $$21 \times 13 = 273$$
- $$21 \times 12 = 252$$
- $$21 \times 11 = 231$$
- $$21 \times 10 = 210$$
- $$21 \times 9 = 189$$
- $$21 \times 8 = 168$$
- $$21 \times 7 = 147$$
- $$21 \times 6 = 126$$
- $$21 \times 5 = 105$$
- $$21 \times 4 = 84$$
- $$21 \times 3 = 63$$
- $$21 \times 2 = 42$$
- $$21 \times 1 = 21$$

Follow up questions

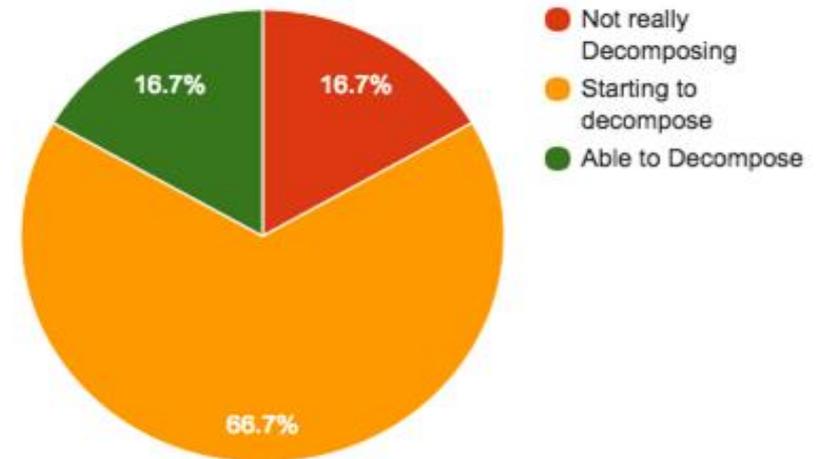
- If you cut up a shape and rearrange the pieces, does the total area change?
- Why might it be useful to cut up a shape to determine its area?
- How else could you cut up your shape to make it easy to relate the area of your shape to the area of another shape with an easy to calculate area?

Pre and Post-Assessment Results

Pre-assessment: Area of Triangle vs. Area of Rectangle (with same base length and height)



Post Assessment Results



DIFFERENTIATED LESSON SURFACE AREA

Stacy Arnold

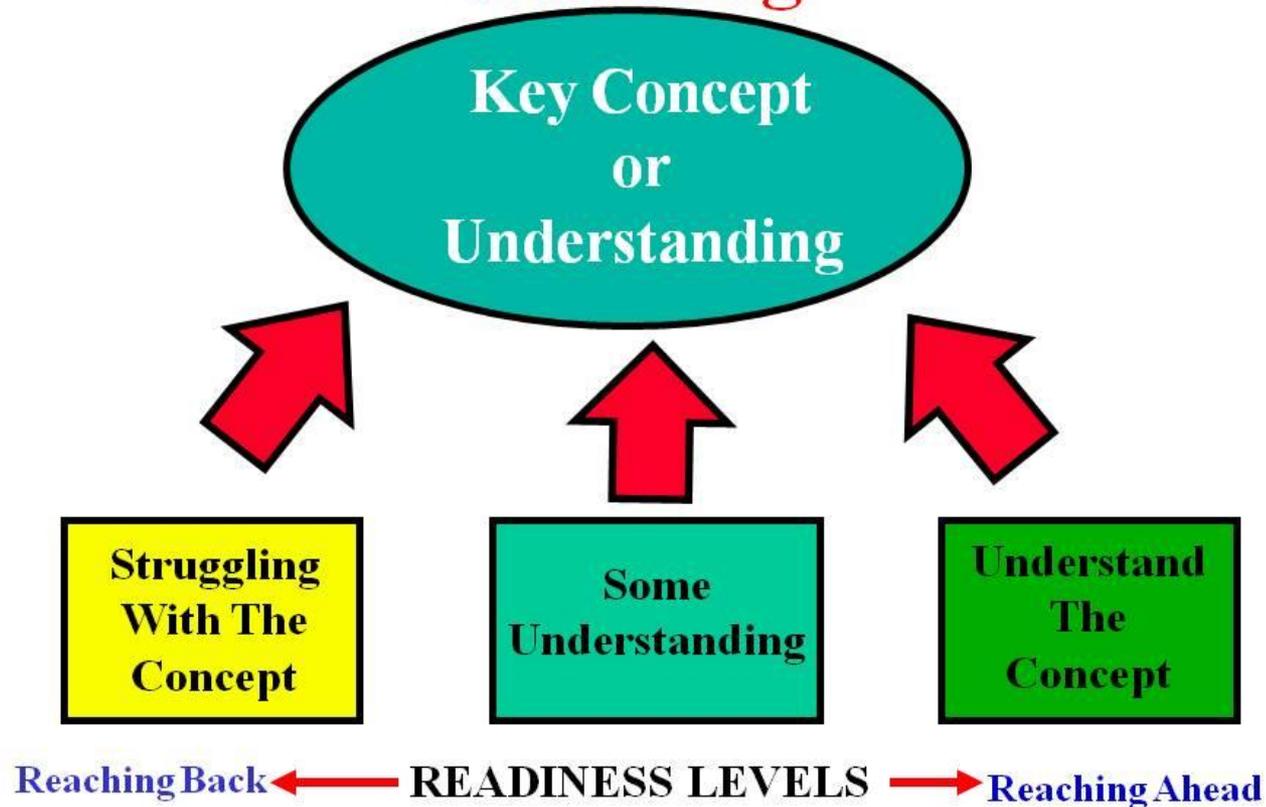
Clay Middle School – Carmel, Indiana

7th Grade Math Class – on grade level

Tiered Instruction based on a Pre-Assessment

Tiered Instruction

Creating Multiple Paths For Learning



Learning Target - Standard

- 7.GM.7: Construct nets for right rectangular prisms and cylinders and use the nets to compute the surface area; apply this technique to solve real-world and other mathematical problems.

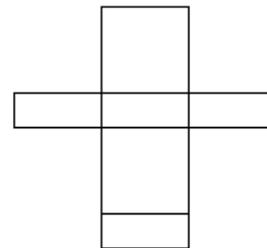


Figure 1

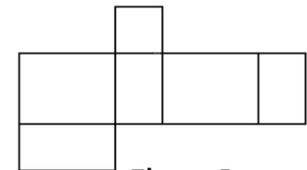


Figure 2

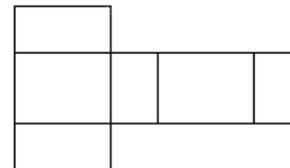


Figure 3

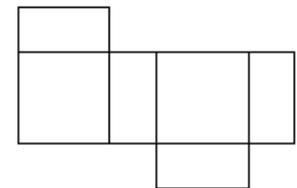
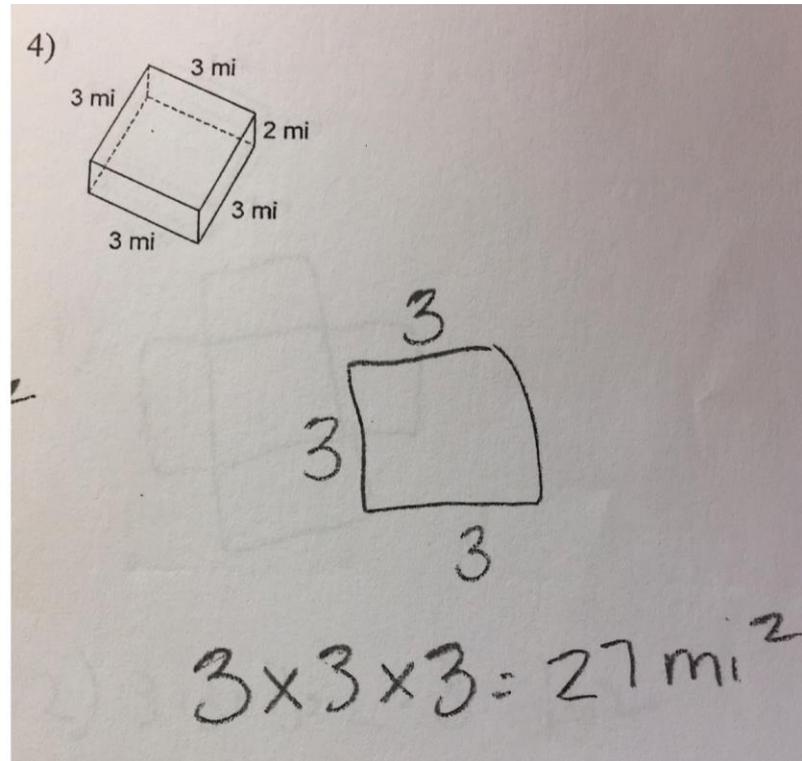


Figure 4

Pre-Assessment – Surface Area

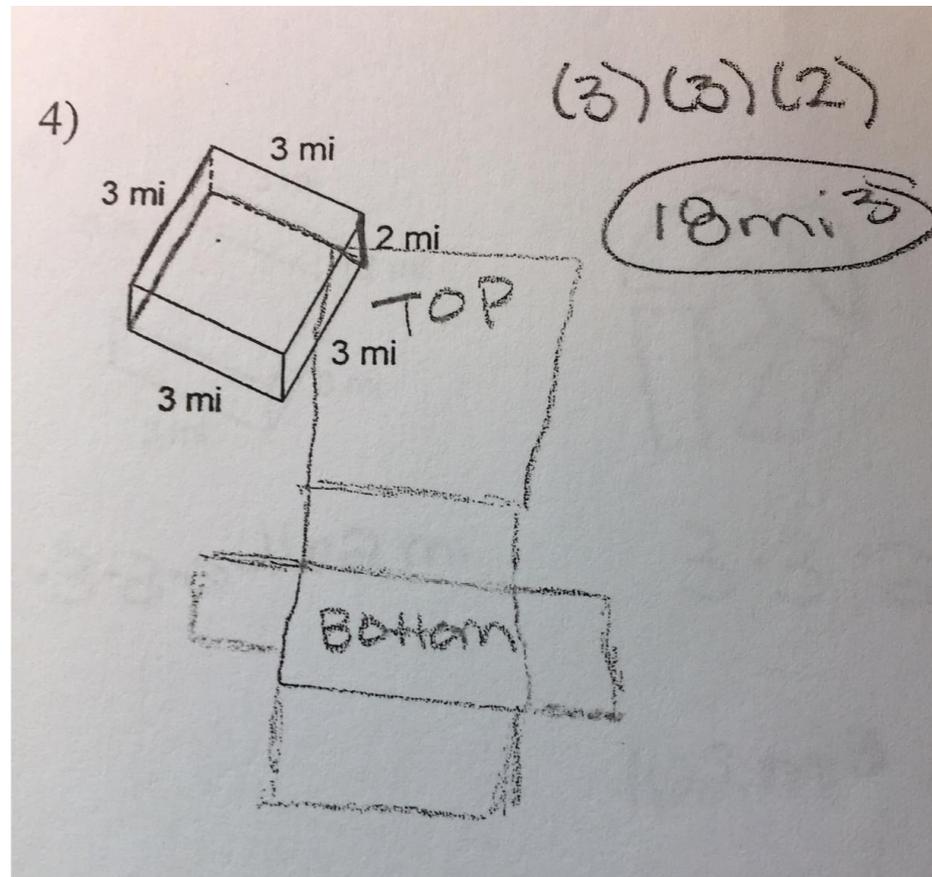
(Students had already completed a unit on volume and had some work with nets.)

- Looked at student work and divided into three groups
 - Group 1 – lowest level of thinking, difficulty drawing nets, surface area calculations incorrect



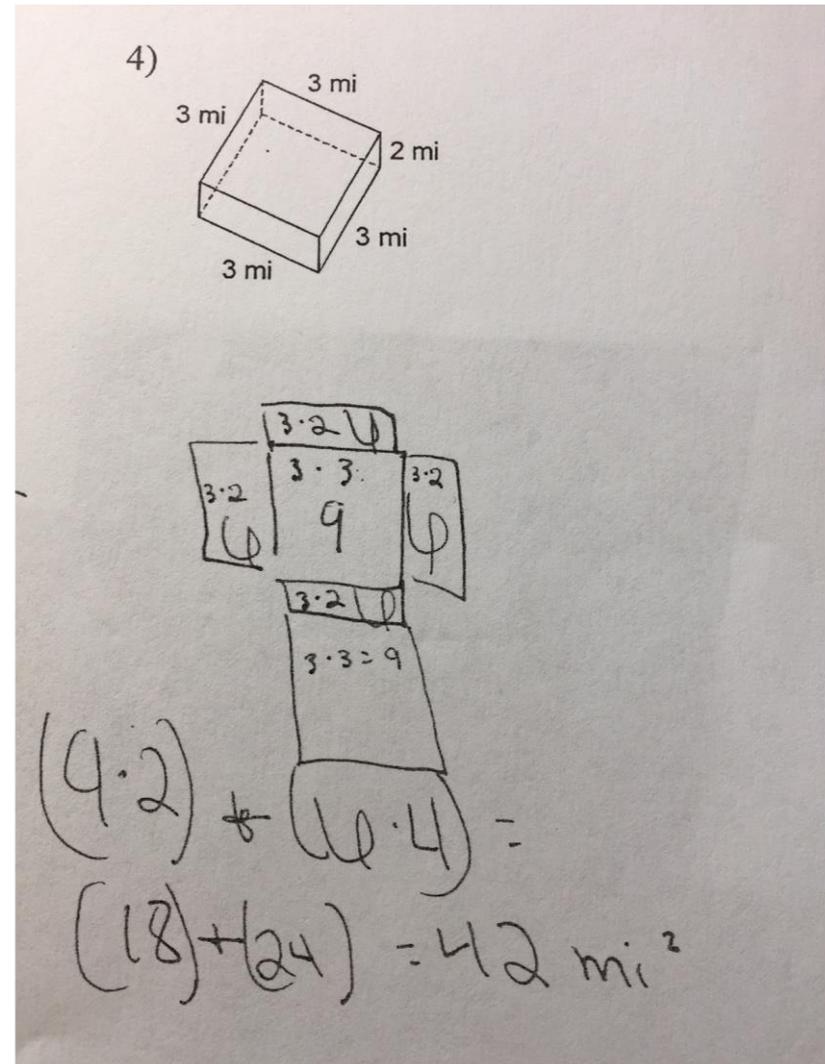
Pre-Assessment — *cont.*

Group 2 – shows some understanding, but not all answers correct, students could draw most nets, could find area on some faces



Pre-Assessment — *cont.*

Group 3 - shows understanding with no or very few mistakes, students could draw all nets, showed work and thinking on each problem



Lesson Plan – Tiered Lesson

- Whole Group Activity – introduce surface area
- **Group 1: Tissue Box Problem** – I wanted to make sure these students could draw a net. They were given a tissue box to measure and cut apart to find the net.
- **Group 2: Cake Pans Problem** – I wanted these students to be able to compare two surface areas and also work with a prism that did not have a typical net. (ex. no top to the cake pan). They had real cake pans to look at for examples, but could not disassemble.
- **Group 3: Box Design** - These students went to the computer lab and worked independently on a slightly more difficult problem. They used the tinkercad program to design two rectangular prisms with the same volume, but different surface areas. They found the price to make each box, and were challenged to design a box using the least amount of plastic.

Homework - Differentiated

- I gave different but comparable homework to each of the groups.
- I used homework from the textbook resources that was already differentiated. (Practice A and B)
- Students compared work and answers in their groups the following day.

Summary

- Pros
 - Students in homogeneous groups based on pre-assessment made it easier to address the needs of the students in groups
 - Tiered instruction seemed effective for meeting students at their readiness level. Need to be prepared to move students to more challenging problems as needed.
 - Advanced students were allowed more independence and were able to go deeper into a topic.
- Cons
 - Time – intensive upfront planning
 - needed more time built into lesson for students to explore problems

DIFFERENTIATED LESSON UNIT REVIEW

Patti Walsh

Tri-North Middle School – Bloomington, Indiana

7th Grade Math Class

Choice Questions and Parallel Tasks

The lesson was about...

- Multi-topic review
- Choice of where
- Choice of who
- Choice of what (topic and difficulty)
- Letting kids self-check and ask for help

MATH TIC-TAC-TOE

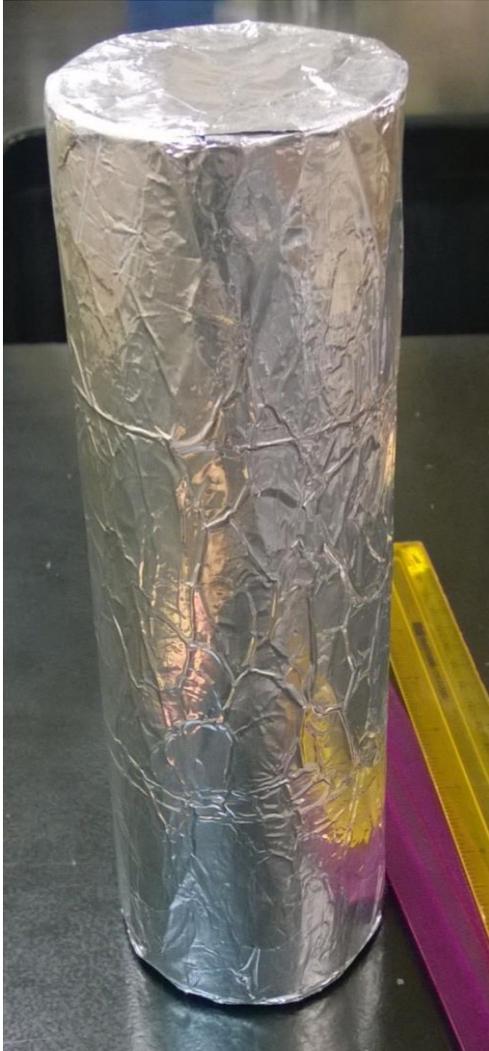
Instructions: Put an "X" through the activities you complete. You must choose activities that give you 3 in a row horizontally, vertically, or diagonally. All the worksheets and files you need are attached to this assignment. Delete any pages you don't use before you submit the file for grading. You must show your work on each activity for credit.

Do 4 problems of your choice from Worksheet #1	Complete the activity at Station #3	Do 4 problems of your choice from Worksheet #2
Complete the activity at	Choose an activity from	Do 4 problems of your choice from Worksheet #3
		Complete the activity at the middle table



the activities you complete for reward dollars

What I knew...



- Some needed procedural practice (pre-built shapes)
- Some needed application practice (at left)
- Everyone was rusty on something (percents)
- We needed a change in our daily routine!

The most popular problem...

A shape, a ruler, and a graduated cylinder.

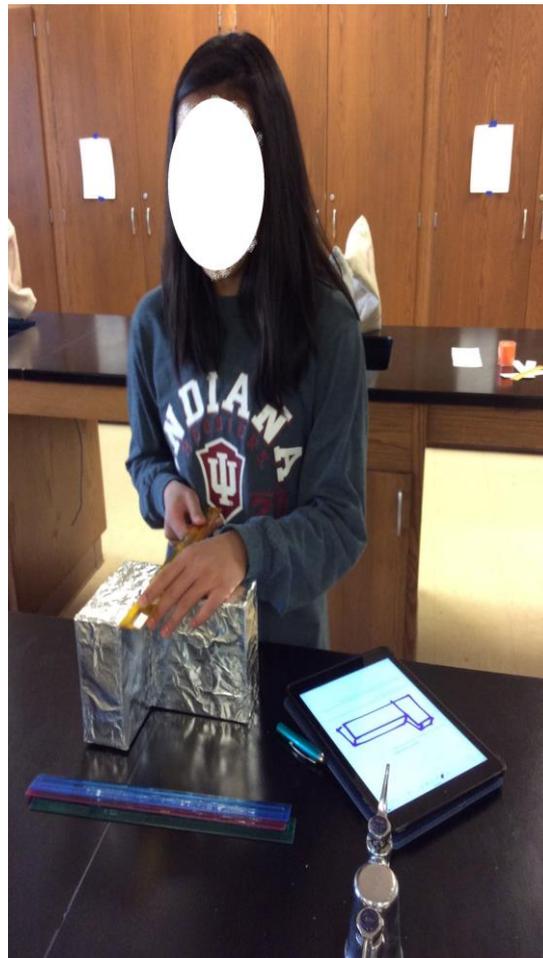
This problem asked student to:

- Measure length
- Calculate volume
- Measure volume
- Calculate percent error

This was harder than many students could handle!



The good...



- Students loved choices
- Students looked forward to movement
- Using this as an extra anchor activity kept the enthusiasm going
- Almost everyone completed the assignment over the course of two weeks
- Lots of students attempted the hardest problem

Examples...

These students asked for guidance on what they should practice

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Anchor Worksheet #2 7.C.6

Date _____ Peri _____

Find each percent change. Round to the nearest tenth of a percent. State if it is an increase or decrease.

1) From 9 hours to 32 hours

$$\frac{32-9}{9}$$
$$\frac{23}{9}$$
$$2.5 \times 100 = 250\%$$

250% Increase

3) From \$73 to \$71

$$\frac{73-71}{71}$$
$$\frac{2}{71}$$
$$0.02 \times 100 = 2\%$$

2% Decrease

2) From 47 tons to 72 tons

$$\frac{72-47}{47}$$
$$\frac{25}{47}$$
$$0.53 \times 100 = 53\%$$

53% Increase

4) From 54 ft to 45 ft

$$\frac{54-45}{45}$$
$$\frac{9}{45}$$
$$0.2 \times 100 =$$

20% Decrease

Find the area of each. Round your answer to the nearest tenth.

1) radius = 2 m

$$A = \pi r^2$$
$$A = \pi (2)^2$$
$$A = 12.6 \text{ m}^2$$

3) diameter = 8 yd

2) radius = 9 cm

$$A = \pi r^2$$
$$A = \pi (9)^2$$
$$A = 254.5 \text{ cm}^2$$

4) diameter = 12 cm

5) circumference = 56.5 ft

6) circumference = 18.8 m

Find the circumference of each circle. Round your answer to the nearest tenth.

7) diameter = 16 in

$$C = \pi d$$
$$C = \pi (16)$$
$$C = 50.3 \text{ in}$$

8) diameter = 13.6 cm

$$C = \pi d$$
$$C = \pi (13.6)$$
$$C = 42.7 \text{ cm}$$

Examples...

Middle Table: Finding Volume and Percent Error

Step 1: Below, write down the measurements in millimeters of the shape at this station that you will need to calculate volume.

50 mm

Step 2: Using the measurements you made calculate the volume of the shape in millimeters cubed.

$$V: L \times h \times w$$
$$50 \times 50 \times 50 = 125,000 \text{ mm}^3$$

Step 3: 1000 mm^3 is the same as 1 mL. Convert your answer from step 2 to milliliters. Write your answer in the box below.

125 mL

Step 4: Show your teacher your work. You will get a graduated cylinder. Using the graduated cylinder, measure the actual volume of the shape with water from the sink. Write the actual volume in milliliters in the box below.

133

Step 5: Percent error is calculated similarly to percent of change:

6.4 mm

$$\text{percent error} = \frac{|\text{estimate} - \text{actual}|}{\text{actual}} \times 100$$

$$\frac{133 - 125}{125} \times 100 = 0.064$$

Using the value you calculated and the value you measured, calculate your percent error

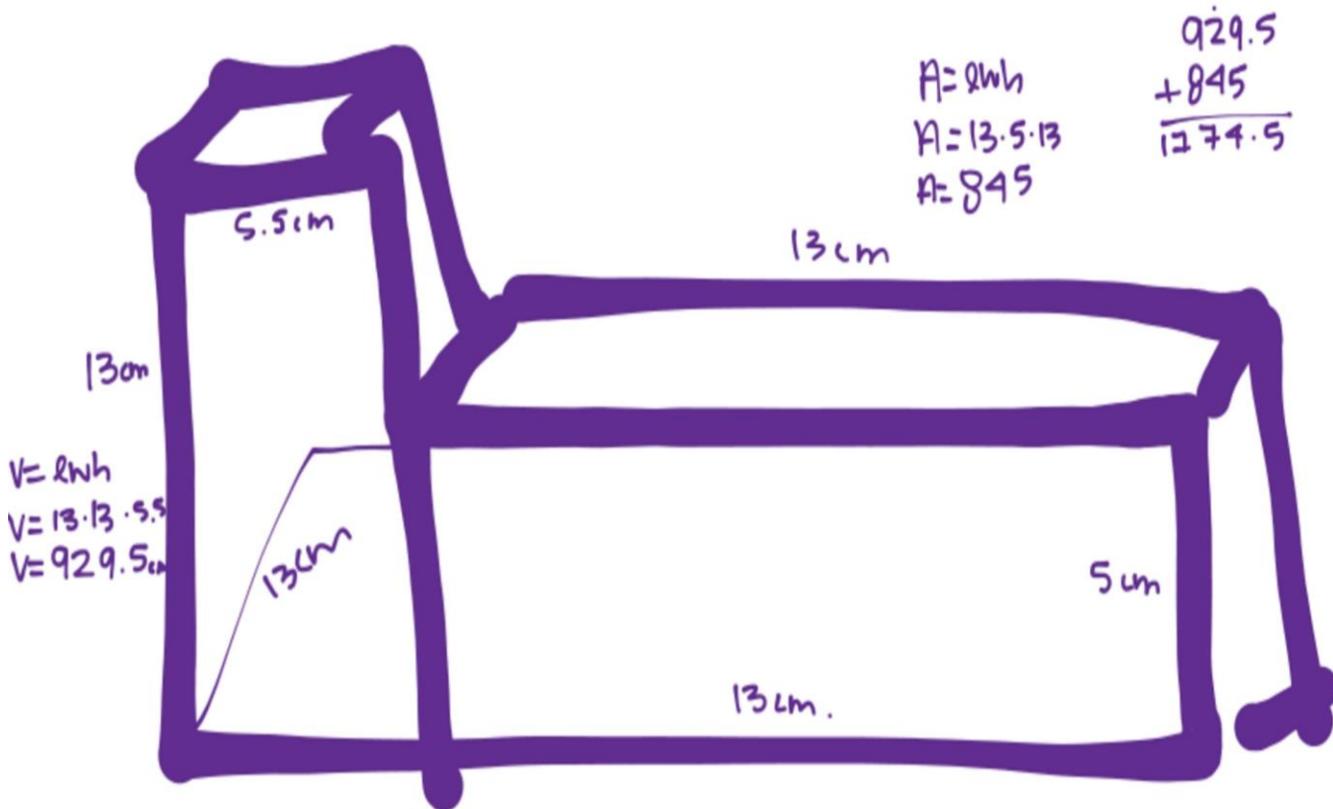
In my view, this student should have chosen the easier stations, but instead (almost) nailed the hardest one!

Examples...

This student chose all the hardest stations

Station #5: Finding Volume

Step 1: Below, draw a diagram of the shape at this station.



The bad...

- Working with friends
- Choosing the easy
- Too much and too little support
- Students did whatever others were doing and didn't read instructions
- We had a lot of absences, **TWO WEEKS**



Examples...

- Without being given all the information, some students weren't sure what to do

Station #6: Finding Volume

Step 1: Below, draw a diagram of the shape at this station.



This student was under-supported mathematically (by me and the problem) and chose collaborators unwisely (this is copied from others' work)

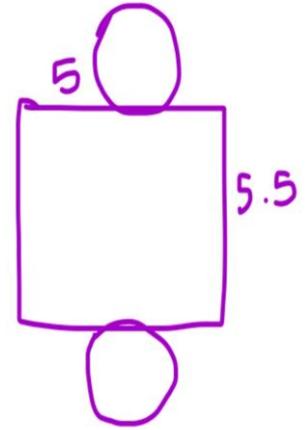
Step 2: Measure the dimensions of the shape in centimeters. Label the diagram you drew with the dimensions you measured.

Step 3: Using the measurements you made and your diagram above, calculate the volume of the shape in centimeters cubed. Put your answer in the box below.

Volume:

$$V = 45$$

Step 1: Below, draw a diagram of the shape at this station.



Step 2: Measure the dimensions of the shape in centimeters. Label the diagram you drew with the dimensions you measured.

Step 3: Using the measurements you made and your diagram above, calculate the volume of the shape in centimeters cubed. Put your answer in the box below.

$$V = l \cdot w \cdot h$$

$$V = 17.5 \cdot 5 \cdot 5 \cdot 5$$

Volume:

$$481.25 \text{ m}^2$$

Examples...

- Vocabulary hampered some students
- There was much confusion between surface area and volume
- This student knew how to calculate volume, but doing it with a real shape threw him off
 - Not useful diagram
 - Not sure which formula

Examples...

Station #8: Finding Surface Area with a Net

Step 1: Below, draw the net of the shape at this station.

$r = 3.9$
 $A = \pi r^2$
 $A = 47.8 \text{ cm}^2$
 $C = 2\pi r$
 $C = 24.5 \text{ cm}$
 $A = C \times h = 24.5 \text{ cm} \times 17.6 = 431.1$

Handwritten notes and calculations:
 2×47.8
 47.8
 47.8
 431.1

Step 2: Measure the dimensions of the shape in centimeters. Label the net you drew with the dimensions you measured.

$$(2 \times 47.8) + 431.1 = 95.6 + 431.1 = 526.7 \text{ cm}^2$$

Step 3: Using the measurements you made and your net above, calculate the surface area

Student knew the idea, but didn't draw the net...

Ideas...

Make menus part of the regular routine by:

- Using anchor activities all the time
- Gradually releasing support
- Mixing and integrating content
- Developing procedures for choosing collaborators
- Being explicit about product expectations

7th Grade Math Name _____ ID: 1
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Anchor Worksheet #4 7.GM.5 Date _____ Period _____

Find the area of each. Round your answer to the nearest tenth.

1) radius = 2 m

2) radius = 9 cm

3) diameter = 8 yd

4) diameter = 12 cm

5) circumference = 56.5 ft

6) circumference = 18.8 m

Find the circumference of each circle. Round your answer to the nearest tenth.

7) diameter = 16 in

8) diameter = 13.6 cm

9) radius = 8 ft

10) radius = 6.9 yd

Final words of advice

- START SMALL!
- Build on what you're already doing; adapt existing materials.
- Assess student thinking! Critical to get to know your students.
 - But don't pigeon-hole students; always give them opportunities to surprise you.
- Find support from others in (or outside) your building you can bounce ideas off.
- Keep the mathematical goal at the center, so you don't get bogged down in irrelevant details.
- Intentionally create a classroom environment that welcomes different ways of thinking.

Discussion Time: Questions? Comments?

Thank you!

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References and Resources

- Heacox, D. (2002). *Differentiating instruction in the regular classroom: How to reach and teach all learners, grades 3-12*. Minneapolis, MN: Free Spirit Publishing.
- Laud, L. (2011). *Using formative assessment to differentiate mathematics instruction, grades 4-10: Seven practices to maximize learning*. Thousand Oaks, CA, and Reston, VA: Corwin and NCTM.
- Small, M., & Lin, A. (2010). *More good questions: Great ways to differentiate secondary mathematics instruction*. New York and Reston, VA: Teachers College Press and the National Council of Teachers of Mathematics.
- Tomlinson, C. A. (2005). *How to differentiate instruction in mixed-ability classrooms* (2nd ed.). Upper Saddle River, NJ: Pearson.
- Carol Tomlinson's website: <http://www.caroltomlinson.com/>